PROBLEMS OF PROBABILITY THEORY

Summary of Lecture given at the meeting of the Moscow Mathematical Society on December 11, 1944

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(Translated by V. V. Sazonov)

I heard about this lecture laying out a Program for the development of Probability Theory more than once from people of the older generation. Recently in the archives in the home of A. N. Kolmogorov a few pages were found (both handwritten and typewritten) containing a summary and a sketch of this lecture. The reader will be interested in getting acquainted with the problems which A. N. Kolmogorov put forward almost half a century ago and which, as time proved, determined the directions of many investigations in Probability Theory and Mathematical Statistics up to our time.

A. Shiryaev

The lecture will contain a characterization of the present state of Probability Theory as well as an attempt to outline the prospects of its development in the coming years. In addition a general characterization of large directions of work which the lecturer believes to be especially important (below they are denoted by Roman numerals), and examples of some precisely stated problems (denoted by Arabic numerals below) will be indicated which deserve the attention of researchers.

I. Axioms and applicability problems.
1. Logical foundation of Mathematical Statistics, i.e., of methods of hypothesis testing, of parameter estimation, of control of mass production by sample observations.
2. Construction of a general theory of observation, connected with the influence of the observer on the observed system, with the aim to find a logical foundation of Quantum Physics.
3. Clarification of the logical essence of probabilistic analogies in Number Theory.

II. Limit Theorems.
4. Refinement of the main classical limit theorem.
5. Limit theorems for distributions in functional spaces as a universal source of special limit theorems of classical type.

III. Infinite-dimensional probability distributions.
6. Distributions of scalar, vector and tensor functions which are invariant with respect to different groups of transformations. This problem is of interest from the point of view of Statistical Mechanics of Continuous Media, in particular, of statistical Turbulence Theory.
7. Distributions of allocations of particles in space which are invariant with respect to different motion groups. The problem is of interest from the point of view of statistical crystal theory and crystallization.

IV. Classical stochastic processes.
8. General solution of the Smolukhovsky equation.
9. Estimation of stationary processes by their values on a bounded interval (parameter estimation and extrapolation problems).
11. Statistical properties of dynamical systems (in the general case).
12. Relations between invertible and noninvertible processes.

It is not the aim of the lecturer to list all the essential problems of related sciences requiring the application of Probability Theory for their solution. Among applied problems we mentioned above only those which are closely connected with the main directions of the development of Probability Theory itself, confining ourselves naturally to the ones for which the statement of the corresponding probability problems is clear to the lecturer. In particular we present the general theory of classical “stochastic processes”, since here, as in the general theory of dynamical systems, an independent purely mathematical direction of investigation has well crystallized itself by now. On the other hand, questions of Quantum Physics appear only in problem 2, since in this area it appears to be difficult to indicate a long range independent direction of a purely mathematical investigation.