The degree of anisotropy for mid-ocean currents from satellite observations and an eddy-permitting model simulation

Huei-Ping Huang¹, Alexey Kaplan¹, Enrique N. Curchitser², and Nikolai A. Maximenko³

H.-P. Huang, Lamont-Doherty Earth Observatory of Columbia University, 61 Route 9W, Palisades, NY 10964, USA. (huei@ldeo.columbia.edu)

A. Kaplan, Lamont-Doherty Earth Observatory of Columbia University, 61 Route 9W, Palisades, NY 10964, USA. (alexeyk@ldeo.columbia.edu)

E. N. Curchitser, Institute of Marine and Coastal Sciences, Rutgers University, New Brunswick, NJ 08901, USA. (enrique@marine.rutgers.edu)

N. A. Maximenko, International Pacific Research Center, SOEST, University of Hawaii at Manoa, Honolulu, Hawaii 96822, USA. (nikolai@soest.hawaii.edu)

¹Lamont-Doherty Earth Observatory of Columbia University

²Institute of Marine and Coastal Sciences, Rutgers University

³International Pacific Research Center, SOEST, University of Hawaii at Manoa
Abstract. The degree of anisotropy is calculated for the mid-ocean currents estimated from satellite altimetry and simulated with a numerical model of the Pacific Ocean. A high resolution eddy-permitting model is used for its ability to simulate mid-ocean multiple zonal flows, crucial for the evaluation of the degree of anisotropy. Using a commonly defined parameter of anisotropy, $\alpha$, that falls between $-1$ and $1$ and equals $0$, $-1$, and $1$ for an isotropic, purely meridional, and purely zonal flow, respectively, it is found that $\alpha$ increases from nearly zero for weekly data to significantly positive values for seasonally and annually averaged data. This tendency of increasing zonal anisotropy with averaging time is true for the surface geostrophic velocity from satellite altimetry and for both surface and deep-ocean velocities from the model simulation. The absolute value of $\alpha$ for the simulated surface currents agree with that derived from satellite observation, reaffirming the reliability of both data sets. In the model simulation, the behavior of $\alpha$ at the surface is very similar to that at 1000 m, indicating a deep structure of the zonally elongated features in the middle of the Pacific Ocean. The implications of these findings are discussed in the context of oceanic eddies, Rossby waves, and zonal jets.
1. Introduction

In the last decade the analyses of high-resolution satellite data have greatly enhanced our knowledge about the spatiotemporal structure of sea surface currents. Using a global data set from satellite altimetry, Maximenko et al. (2005) showed that the geostrophic currents derived from the observed sea surface height exhibit multiple zonal-jet structures upon time averaging, consistent with recent discoveries of mid-ocean jets in eddy-permitting ocean model simulations (Nakano and Hasumi 2005, Richards et al. 2006). These recent findings raise many interesting questions, the most basic of which concerns the realism and robustness of the multiple zonal “stripes” in the velocity field. In recent studies the discussion on mid-ocean jets is often based on the map of the zonal component of the velocity, $u$, or that of the geostrophic velocity, $u_g$. Given that $u_g$ is proportional to the meridional derivative of sea surface height (SSH), $h$, a random isotropic field of $h$ could already correspond to east-west banded structures in $u_g$ even when no real zonal jets exist. In this case, the meridional geostrophic velocity, $v_g$, would exhibit north-south stripes since $v_g$ is proportional to the zonal derivative of $h$. (See Appendix A.) The flow field is not truly zonally anisotropic if the north-south stripes in $v_g$ are as strong as the east-west stripes in $u_g$. There is indeed a hint of this situation in a typical example, shown in Fig. 1, of a set of weekly fields of $u_g$ and $v_g$ over the North Pacific derived from satellite altimetry (detail in Section 2). Figure 2 is similar to Fig. 1 but for 50-week averaged fields of $u_g$ and $v_g$. Here, after substantial time averaging, the east-west stripes in $u_g$ become much stronger than the (still visible) north-south stripes in $v_g$, indicating that zonal anisotropy is real on the 50-week time scale. (Alternatively, one can also show
that the zonal jets are real by demonstrating their existence in the vorticity field, as did Maximenko et al. (2005.)

The examples in Figs. 1 and 2 suggest two points for investigations. First, to solidify the claims made in recent studies of the dominance of multiple zonal flows in the mid-ocean, it is useful to consider both $u$ and $v$ components and quantify the degree of anisotropy of the velocity field. Secondly, the degree of anisotropy and the zonal-jet structure of the flow field clearly depends on time averaging. For example, Maximenko et al. (2005) suggested that time averaging of westward drifting eddies may lead to a visually zonal-jet like structure. Thus, the time scale at which zonal anisotropy emerges may provide a useful hint for the dynamics of the zonally coherent structures. Anticipating future investigations in this direction, in this study we will perform straightforward calculations of the degree of anisotropy for the velocity field as a function of the length of time averaging for both observations and a numerical simulation.

Observations from satellite altimetry and a simulation based on an eddy-permitting model for the Pacific are used in this study. Although sparse in-situ observations exist for the surface and deep ocean currents, we choose the altimetry data for its extensive and uniform spatial-temporal coverage at high resolution that are needed for our calculation. Satellite data only provide information for the surface. They are complemented by the ocean model simulation that produces both surface and deep ocean currents. A cross-validation of the simulated and observed currents at the surface will help reassure the reliability of both data sources. The model used here is eddy-permitting, guided by results from recent studies that mid-ocean multiple zonal flows emerge only after the model properly resolves mesoscale eddies (Treguier et al. 2003, Nakano and Hasumi 2005). Our
comparison of the statistics of the surface currents from high-resolution satellite data and eddy-permitting model simulation is among the first of its kind. The degree of anisotropy provides a simple measure of the “zonality” of the flow field, ideal for a first quantitative analysis and comparison of the strength of zonal currents in observations and simulations. Considerations of more complicated statistics, e.g. those that measure the sharpness or curvature of the profile of zonal velocity, are left for future work. In the following, Section 2 is devoted to the calculations for the degree of anisotropy for the altimetry data. Section 3 repeats these calculations for the numerical simulation. Discussion and concluding remarks follow in Section 4 and 5.

2. Satellite observation

2.1. The altimetry data

The data used here is from the processed AVISO altimetry measurements for SSH (Ducet et al. 2000). It has 1/3-degree Mercator resolution in space and is archived as weekly means. Since the AVISO data was processed with a mapping function with a 15-day time scale, it is understood that the “weekly fields” in this paper are in fact temporally smoothed with an effective resolution slightly longer than a week. The coverage of the data is global but for our purpose it suffices to focus on the North Pacific.

The surface geostrophic velocities, \((u_g, v_g)\), are derived from SSH by the geostrophic relationship, taking into account spherical geometry. A local fourth order finite difference scheme (e.g., Castillo et al. 1995) is used. Since geostrophic relationship breaks down in the lower latitudes and since the Tropics has its own distinctive current system, only the domain north of 12°N (the entire domain shown in Fig. 1) is considered. The weekly data consists of 639 records spanning about 12 years. Guided by Figs. 1 and 2, N-week
averaged data sets (based on non-overlapping N-week segments, e.g., there are 127 such segments for \( N = 5 \)) are constructed from the weekly data, with selected values of \( N = 1, 5, 10, 20, 50, \) and 100. (\( N = 1 \) corresponds to the “unaveraged” weekly data.)

The altimetry data used here is the anomaly relative to the long-term (1992-2002) mean. The long-term mean of the altimetry SSH includes geoid that is unrelated to geostrophic ocean currents. The construction of the absolute dynamic topography based on different combinations of satellite and in-situ data is by itself a complicated issue (e.g., Tapley et al. 2003, Maximenko and Niiler 2005, Jayne 2006). We will first focus on the anomalous SSH in the analysis of the satellite data, deferring a discussion on the absolute velocity to later sections.

In addition to the long-term mean, the mean seasonal cycle, defined as the sum of the averaged annual and semiannual harmonics, is also removed from the SSH in our major calculations for both satellite observations and the model simulation. We have tested selected cases to ensure that the removal or addition of the seasonal harmonics does not affect our conclusions.

### 2.2. Degree of anisotropy

The degree of anisotropy of the horizontal velocity for a given spatial domain is defined by

\[
\alpha = \frac{\langle u^2 \rangle - \langle v^2 \rangle}{\langle u^2 \rangle + \langle v^2 \rangle},
\]

where \( \langle u^2 \rangle \) indicates the area-weighted domain average of \( u^2 \). The domain averaged velocity is removed before the calculation of the variance. For the satellite data, geostrophic velocities \( (u_g, v_g) \) are used in the places of \( (u, v) \) in (1). The definition in (1) is standard.
and has been used elsewhere for the studies of quasi-two-dimensional flows (e.g., Shepherd 1990). Note that $-1 \leq \alpha \leq 1$ and $\alpha = 0$, $-1$, and 1 for an isotropic, purely meridional (north-south), and purely zonal (east-west) flow, respectively. The value of $\alpha$ is connected to the horizontal aspect ratio of the disturbances in the flow field. Consider for instance a disturbance with $\psi \sim \exp[i(k_x x + k_y y)]$ in a non-divergent flow where $\psi$ is the stream function and $k_x = 2\pi / L_x$ and $k_y = 2\pi / L_y$ are horizontal wavenumbers. Using the relation, $(u, v) = (-\partial\psi/\partial y, \partial\psi/\partial x)$, and (1) we obtain

$$\alpha = \frac{1 - (L_x/L_y)^{-2}}{1 + (L_x/L_y)^{-2}} \quad (2)$$

Thus, $\alpha \to 1$ when $L_x \gg L_y$, and $\alpha \to -1$ when $L_x \ll L_y$.

The values of $\alpha$ are calculated for the whole North Pacific (north of 12°N) domain and for several of its sub-domains, shown in Fig. 2b as the boxes. They include Box 1 in the Eastern Pacific far away from the western boundary currents (but including some eastern boundary currents), Box 2 to its west that contains some of the extensions of the western boundary currents, and Box 3 in the far North Pacific, an area that includes significantly non-zonal bottom topography and boundary currents.

Figure 3 shows the value of $\alpha$ for the four named domains as a function of $N$, the number of weeks over which time averaging is performed. The filled circles are the mean. For example, for $N = 5$ it is the average over 127 values of $\alpha$ each calculated with Eq.(1), with the $u_g$ and $v_g$ in the equation being the 5-week means. The vertical bar indicates ±1 standard deviation. (The vertical bar for $N = 100$ might be less reliable since it is deduced from only 6 records.) Two important features emerge. First, except for Box 3, the unaveraged weekly surface geostrophic current is nearly isotropic with $\alpha \approx 0$. This confirms our first impression of Fig. 1 as described in Section 1. Second, $\alpha$ clearly
increases toward positive zonal anisotropy with time averaging. This tendency is true for all domains. The typical value of $\alpha$ is between 0.3 and 0.5 under a 1-year average depending on the location. The lower value of $\alpha$ for Box 3 may be due to the permanent anisotropic signatures in the flow in that region associated with the Aleutian boundary current. Note as well that in the higher latitudes the eddies are generally smaller due to a larger Coriolis parameter and, hence, a smaller deformation radius (Chelton et al. 1998). They might be relatively poorly resolved by altimetry (e.g., Pascual et al. 2006). The largest (positive) values of $\alpha$ are found in Box 1, far away from the influences of the western boundary currents.

3. Ocean model simulation

3.1. The model

The ROMS (Regional Ocean Modeling System) model (Curchitser et al. 2005, Shchepetkin and McWilliams 2003) is used for the eddy-permitting simulation of the Pacific Ocean. It has a flexible horizontal grid system and terrain-following vertical levels. This study uses a new run with a 0.18 degree horizontal resolution and 42 terrain-following levels. An illustration of the model levels and detail of the bathymetry are given in Appendix B. The model domain extends from 30S to 65N, and from 90E to 290E. The surface forcing is derived from the CORE data set (Large and Yeager, 2004). The high-resolution North Pacific model is nested within a 1 degree global ocean hindcast simulation using the National Center for Atmospheric Research (NCAR) Community Climate System Model (CCSM). The initial conditions are derived from a fully evolved ocean model so that a relatively short period of adjustment is needed. For the comparison with altimetry data, we will focus on the North Pacific domain between 12N and 60N. The model outputs
from the last 12 years, comparable in length to the satellite data, of a 20-year run will be used. Unlike the satellite data, the model simulation produces a straightforward long-term mean of the velocity field. For a comparison with satellite data we will first analyze the anomalies but will restore the long-term mean in a later discussion.

Figure 4 shows the model simulated surface velocity. (Hereafter, the velocity from the model refers to the full velocity, not geostrophic velocity derived from the SSH field as in the satellite data.) Figure 4a is the long-term (12-year) mean of $u$, Fig. 4b shows a selected one-year mean of $u$, and Fig. 4c is the one-year mean of the anomaly of $u$, defined as Fig. 4b minus Fig. 4a. Figure 4d is similar to Fig. 4c but for the meridional velocity, $v$. Now Figs. 4c and 4d can be compared with the 50-week averaged geostrophic velocity in the satellite data in Figs. 2a and 2b. They contain similar features, with the east-west stripes in $u$ much stronger than the north-south stripes in $v$, indicating the dominance of zonal velocity under the annual average. The magnitude of the simulated surface velocity is also similar to its counterpart in satellite data. Note that the east-west stripes in $u$ are also visible in the one-year mean in Fig. 4b and even the long-term mean in Fig. 4a, although they are not as prominent as those in the anomaly field in Fig. 4c.

3.2. Degree of anisotropy at the surface

We first evaluate $\alpha$ for the simulated anomalous surface velocity for a direct comparison with the satellite observations. The statistics of $\alpha$, the counterparts of Fig. 3 for the model simulation, are shown in Fig. 5. Because the model outputs were archived as four-day means, the previously used 1, 5, 10, 20, 50, and 100 weeks of time averaging are approximated as 8, 36, 72, 140, 348, and 700 days, respectively, for the model data. For simplicity we retain the notations in Fig. 3. Figure 5 agrees with Fig. 3 in the overall
values of $\alpha$ and their tendency to increase with time averaging. The agreement extends to individual subdomains of the North Pacific. For example, Box 1 in the Eastern Pacific has the highest values of $\alpha$, while Box 3 has the lowest values with $\alpha$ being slightly negative at $N = 1$. In both model and satellite data, over the North Pacific and Boxes 1 and 2, $\alpha$ is close to zero for $N = 1$, confirming that the unaveraged weekly velocity field is nearly isotropic.

3.3. Degree of anisotropy at 1000 m

The model simulation provides additional information of the flow fields in the deep ocean. Figure 6 shows the counterparts of Fig. 4 for the model-simulated velocity at 1000 m. The east-west stripes in $u$ are even more prominent in the deep ocean compared to those at the surface. The zonal bands are clearly visible not only in the anomaly field (Fig. 6c) but also in the total $u$ field (Fig. 6b) with one-year averaging. These are consistent with other recent eddy-permitting simulations of the Pacific and other ocean basins using different models (Nakano and Hasumi 2005, Maximenko et al. 2005, Richards et al. 2006, Treguier et al. 2003) In the long-term mean (Fig. 6a), the multiple zonal bands are relatively weaker in the North Pacific but they remain visible. A comparison of Figs. 6c ($u$ velocity) and 6d ($v$ velocity) indicates that the flow in the mid-ocean is dominated by the zonal velocity. This may justify the focus on the $u$ velocity in identifying the deep ocean zonal-jet structure in previous modeling studies (Nakano and Hasumi 2005, Richards et al. 2006).

Figure 7 shows the values of $\alpha$ for the simulated velocity at 1000 m in the same format as Fig. 5. The behavior of $\alpha$ in the deep ocean is similar to that at the surface. Again, $\alpha$ increases with time averaging, with its highest values associated with Box 1 and lowest
values with Box 3. Except for Box 3, the values of $\alpha$ at 1000 m generally exceed those at the surface. Notably, for the North Pacific and Boxes 1 and 2, $\alpha$ is already positive even for $N = 1$ (the weekly data).

3.4. The Tropics

We have so far excluded the Tropics from our discussion, partly to circumvent the inaccuracy in the geostrophic velocity derived from the satellite-observed SSH but also due to the consideration that the zonal currents in the Tropical ocean are governed by dynamics different from that governing the extratropical mid-ocean currents of our interest. Nevertheless, since east-west stripes in $u$ are clearly visible at low latitudes in the model simulation, for completeness of this work, $\alpha$ is calculated from the simulated velocity at the surface and at 1000 m as shown in Fig. 8. Here, the Tropical region is defined as the latitude belt between 18N and 18S. Large positive values of $\alpha$ are found both at 1000 m and at the surface. They are greater than those for the midlatitude shown in Figs. 5 and 7.

3.5. The long-term mean

The preceding discussions have excluded the long-term mean of the velocity field. With the satellite data, the determination of the absolute dynamic topography is non-trivial because the time-mean SSH from altimetry contains elements (geoid, tides) that are unrelated to geostrophic ocean currents. Recently, attempts have been made to construct the long-term mean absolute dynamic topography from different combinations of altimetry, GRACE, and in-situ observations (e.g., Tapley et al. 2003, Maximenko and Niiler 2005, Jayne 2006). The geostrophic $u$-velocity derived from a 10-year (1992-2002) mean of
the absolute dynamic topography with 0.5° resolution from Maximenko and Niiler (2005, which combines all of the altimetry, GRACE, NCEP wind, and in-situ drifter data) is shown in Fig. 9. In the North Pacific, Fig. 9 broadly resembles the simulated long-term mean of the surface $u$-velocity in Fig. 4a, both in magnitude and in large-scale pattern. At the finer scales, there is a hint of multiple zonal bands in the observation but they are not as prominent as those in the model simulation.

Unlike the observation, the ocean model simulation produces a straightforward time-mean velocity field as shown in Figs. 4a and 6a. Adding the time mean back to the anomaly fields, the statistics of $\alpha$ for the total velocity at the surface and 1000 m are calculated for the North Pacific as shown in Figs. 5a and 7a in open circles. In general, up to $N = 50$, the values of $\alpha$ and their tendency to increase with time averaging remain similar with or without the time mean.

4. Discussion

We have shown that a 20-to-50 week average generally leads to significantly positive zonal anisotropy of the velocity field at both surface and 1000 m. This finding supports the uses of 18-week to multi-year averaged $u$ field in identifying the mid-ocean zonal jets in recent observational and modeling studies (Maximenko et al. 2005, Richards et al. 2006). The unaveraged velocity field is found to be nearly isotropic. This may reflect the direct influences of high-frequency random atmospheric forcing. Notably, at the weekly time scale, the velocity at the surface is especially close to isotropy while that at 1000 m – away from the atmospheric influence – is slightly zonally anisotropic.

The time scale we found on which zonal anisotropy emerges is also consistent with an early analysis by Cox (1987), who considered the structure of the stream function (equiv-
alent to our use of both $u$ and $v$ components) simulated by an idealized eddy-permitting model. The temporally unfiltered stream function field is nearly isotropic, while strongly zonal anisotropy emerges under a temporal low-pass filter with a decorrelation scale of about a half year (see Figs. 12 and 13 of Cox 1987).

A deeper understanding of the behavior of $\alpha$ depicted in Figs. 3-7 would require detailed studies based on the dynamical properties of the oceanic eddies/waves and jets. To stimulate future research, a few (not all-inclusive) scenarios are outlined in the following.

4.1. Time averaging of longitudinally drifting eddies

Maximenko et al. (2005) have previously noted that averaging a westward drifting eddy over time may produce a visually jet-streak like structure, corresponding to a positive $\alpha$. However, they also found that the characteristic length scale of an observed zonal jet streak generally exceeds that of the expected displacement of the eddies (Chelton and Schlax 1996). For the current discussion, it is interesting to derive $\alpha$ as a function of the averaging time, $T$, from a simple setting that mimics a field of drifting eddies.

Assuming a constant east-west drifting velocity, $V$, and an initially isotropic field of eddies with a spatial scale $L_x = L_y = R$. Averaging over time, $L_y$ remains unchanged but the drifting leads to $L_x = R + VT$. Using Eq. (2), one obtains

$$\alpha = \frac{1 - (1 + \gamma T)^{-2}}{1 + (1 + \gamma T)^{-2}}$$

where $\gamma = V/R$ is a constant. For example, choosing $R = 50 \ km$ (typical for the mesoscale eddies) and $V = 1 \ cm/s$ gives $\gamma^{-1} \sim 8 \ weeks$. Figure 10a shows selected curves of $\alpha(T)$ with $\gamma^{-1} = 8, 16, 32, \ and \ 64 \ weeks$. Qualitatively, these curves exhibit some of the characteristics of the observed $\alpha$, with $\alpha = 0$ at $T = 0$ and $\alpha$ increases with $T$. However,
superimposing over these curves the values of $\alpha$ from the observation and ocean model simulation (symbols in Fig. 10a) does not reveal a good fit of the data. The goodness of fit can also be measured by the extent that $\gamma$ remains a constant (independent of $T$) for the observed or simulated data. Using the relation

$$\gamma = T^{-1}\left(\frac{1+\alpha}{1-\alpha}\right)^{1/2} - 1$$

derived from Eq. (3), the values of $\gamma(T)$ for selected data from Figs. 3-7 are plotted in Fig. 10b. They do not remain constant but vary by an order of magnitude for the range of $T$ from 0-100 weeks.

This example only considers the simplest setting, with a strictly east-west drift velocity (ignoring the expected slight north-south drift and the cyclone-anticyclone asymmetry, Morrow et al. 2004; Chelton et al. 2006) and a fixed shape of the eddies as they drift. Nevertheless, it illustrates that it is not trivial to explain the zonal anisotropy by considering only a simple drift of the eddies. More complicated mechanisms may be needed to quantitatively explain the observed and simulated jet streaks.

4.2. The non-isotropic dispersion relation of Rossby waves

In contrast to the “drifting eddy” point of view, one may consider another limit when the ocean is occupied by approximately linear waves that preserve their dispersion relation. The linear wave thinking is often adopted for studying the large-scale disturbances in the atmosphere. To draw an analogy, note that a similar behavior of an increasing $\alpha$ with $T$ is known for the synoptic and low-frequency variability in the atmosphere. High-frequency (e.g., $T \sim 1$ week) disturbances tend to be meridionally elongated while low-frequency (e.g., $T > 1$ month) ones are zonally elongated (Wallace and Lau 1985). Because large-
scale, low-frequency disturbances in the atmosphere are dominated by Rossby waves and
are nearly barotropic, one could possibly interpret this behavior as a consequence of the
dispersion relation of the barotropic Rossby wave modes, \( \omega_k = -\beta k_x/(k_x^2 + k_y^2) \), where \( \omega_k \) is the frequency of the \( k \)-th mode, \( k = (k_x, k_y) \) is the wavenumber vector, and \( \beta \) is the meridional gradient of the Coriolis parameter, \( f \). For a given total wavenumber, \( K = (k_x^2 + k_y^2)^{1/2} \), zonally elongated disturbances \( (k_x < k_y, \text{a special case is the zonal mean flow with } k_x = 0) \) are more capable of surviving a low-pass temporal filter because they have lower frequencies. (For simplicity, we ignore the effect of the climatological mean flow on \( \omega_k \), which is important for the atmosphere). Thus, the value of \( \alpha \) for the wave field increases after time averaging.

The above argument relies on the fact that the dispersion relation of Rossby waves is non-isotropic, in the sense that \( \omega_k \) is not invariant under a swapping of \( k_x \) and \( k_y \). Quantitatively, it is complicated to test this idea for the ocean, due to the presence therein of many different wave modes (barotropic and baroclinic Rossby, gravity, etc.) that possess different dispersion relations. As a pedagogical example, in Appendix C we illustrate the increase of \( \alpha \) with \( T \) for a random wave field that consists of only the first baroclinic Rossby wave modes. In that model, the value of \( \alpha \) at large \( T \) turns out to be not unreasonable compared to the observation, although \( \alpha \) is found to be somewhat sensitive to the choice of the prescribed energy spectrum of the unaveraged wave field.

### 4.3. Dynamical pictures and beyond

The scenarios discussed above represent the two limits of idealization (eddies that preserve their shape vs. waves that preserve their dispersion relation). The real world is likely located somewhere in between. Nevertheless, the simple examples serve to illustrate that,
in principle, a theory for the mid-ocean currents or eddies/waves should be able to predict the behavior of $\alpha$, which can then be verified with those obtained from the observations or comprehensive model simulations.

The simple models discussed in Section 4.1 and 4.2 are kinematic, disregarding how the ensemble of waves or eddies are generated or how they interact among themselves. A more comprehensive theory should ultimately include this aspect by incorporating relevant dynamical processes for the eddies and waves. An example of such processes is the upscale energy cascade in geostrophic turbulence (e.g., Smith and Vallis 2001, Galperin et al. 2004, Scott and Wang 2005 for recent discussions) that takes place between the Rossby radius of deformation, $L_R \sim 50$ km, where the mesoscale eddies are generated, and Rhines scale, $L_\beta \sim$ a few hundred km, that coincides with the meridional scale of the jet streaks (Rhines 1975, 1979). While it is beyond our scope to test this dynamical picture, it is useful to note that, for Rossby wave turbulence, the triad interaction that forms the basis of the up-scale energy cascade inherently implies a tendency toward zonal anisotropy by Hasselmann’s lemma (see Rhines 1975, Fu and Flierl 1980, Huang et al. 2001). In short, it dictates that energy be transferred from high frequency to low-frequency modes but the latter have stronger zonal anisotropy for Rossby waves. Thus, the values of $\alpha$ shown in Figs. 3, 5, and 7 might also be used as a constraint to verify a proposed dynamical process for the formation of jet streaks in mid-ocean.

Lastly, we note that several recent studies have also attempted to interpret the anisotropic property of the observed sea surface disturbances from different perspectives. Maximenko and Niiler (2006) noted that the mid-ocean jet streaks are not always strictly zonal but they often have a tilt. The tilted structure is interpreted as stationary Rossby
waves standing in the large-scale background flow. (In the context of our analysis, stationary Rossby waves by definition will survive time averaging for any values of $T$.) Logan et al. (2006) and Sen et al. (2006) found practical isotropy in the weekly surface geostrophic velocity from altimetry but they also suggested the existence of persistent small-scale anisotropic structures in the flow field that are independent of time averaging. How to reconcile and unify all of the above point of views is a major task for future studies.

5. Concluding remarks

Our analysis of $\alpha$ provides a quantitative support for the claimed basin-wide "zonality" (dominance of zonal velocity in the flow field) in the surface and deep ocean currents based on 18-week to multi-year averaged $u$ velocity (Maximenko et al. 2005, Richards et al. 2006). On these time scales, the degree of anisotropy is found to be significantly positive and similar in value in both the surface velocity field from satellite altimetry and the surface and deep ocean velocity fields from an eddy-permitting model simulation. The unaveraged weekly velocity field is found to be nearly isotropic. The effect of low-pass temporal filtering is to remove meridionally elongated structures but spare or accentuate zonally elongated ones. The connection of this behavior with the property of Rossby waves (with or without a background mean flow) or that of a set of drifting eddies awaits further investigations.

Our results also confirm, for the first time, the consistency in the anisotropic properties of the velocity fields between high-resolution satellite observations and an eddy-permitting ocean model simulation. These results raise our confidence in using the ocean model for future investigations of mid-ocean jets, whose basin-wide three-dimensional structures are difficult to observe by either conventional or satellite-based methodologies. Our analysis
of the model output reveals very similar behaviors of $\alpha$ at the surface and at 1000 m, hinting at possibly a deep structure of the zonally anisotropic features in mid-ocean.

The degree of anisotropy calculated in this study measures the zonality of the flow field regardless of the detailed profiles of the zonal velocity. Analyses of other refined measures, for instance one that detects the sharpness of the velocity profile (such a measure would have to depend on the gradient of the velocity), are useful next steps. Nevertheless, the simple definition of $\alpha$ here allows a quick calculation of its value for any flow field from observations, simulations, and theories. As a first test, we expect a viable theory for the mid-ocean zonal currents to be able to reproduce the behavior of $\alpha$ as shown in Figs. 3, 5, and 7.

Appendix A: Some kinematic relationships concerning isotropy

Consider a random isotropic field of SSH with $|h_k|^2 \sim K^{-p}$, where $h_k$ is the $k$-th Fourier component of $h$, $k = (k_x, k_y)$ is the wavenumber vector, and $K = (k_x^2 + k_y^2)^{1/2}$ is the total wavenumber. (In the ensuing argument, $K^{-p}$ can also be replaced by a more general function, $H(K)$, that depends only on the total wavenumber.) By using the geostrophic relation between $u$ and $h$ in spectral space, $u_k \sim ik_yh_k$, the “energy spectrum” for the $u$-component of geostrophic velocity becomes $|u_k|^2 \sim k_y^2K^{-p}$, which is not isotropic but has more energy in the Fourier components with $|k_y| > |k_x|$ (those with a zonally elongated structure) than in those with $|k_x| > |k_y|$. The opposite is true for the $v$-component of the geostrophic velocity, with $|v_k|^2 \sim k_x^2K^{-p}$. However, putting $u$ and $v$ together, the energy spectrum for the velocity vector field $(u, v)$ is isotropic with $|u_k|^2 + |v_k|^2 \sim K^{-p+2}$. Similarly, the energy spectrum of the stream function or vorticity field constructed from the velocity vector is also isotropic.
Appendix B: ROMS model domain and bathymetry

The model domain for the Pacific run used in Section 3 is shown in Fig. 11. Although our analyses are focused on the North Pacific, the model covers part of the South Pacific and the entire Tropical Pacific. A realistic bathymetry is used, as also shown in Fig. 11. Note that the choices of the bathymetry (realistic vs. flat-bottom) may affect the temporal behavior of the simulated zonally elongated structures in mid-ocean, as demonstrated by Nakano and Hasumi (2005). The model has adjustable, terrain-following, vertical levels. For the model run used in this study, 42 vertical levels are used. Fig. 12 illustrates the distribution of the vertical levels along the cross section at 40N. The velocity fields at 1000 m analyzed in Section 3 are obtained by a cubic-spline interpolation in the vertical.

Appendix C: Increase of $\alpha$ by temporally averaging a Rossby wave field

As discussed in Section 4.2, time averaging can affect the degree of anisotropy of a wave field if the dispersion relation of the waves is non-isotropic. Quantifying this effect for the ocean is not trivial due to the presence of many different types of waves with different dispersion relations. Just for demonstration, here a simple example is given by considering a random field of only the first baroclinic Rossby modes with an isotropic spectrum, $|\psi_k|^2 \propto K^{-p}$, where $\psi$ is the baroclinic stream function. We use $p = 3$ and 3.5, values that are arbitrarily chosen but partly inspired by the considerations that (i) In midlatitude with a sharp upper-layer stratification, the baroclinic stream function contributes significantly to the surface stream function (Smith and Vallis 2001, Scott and Wang 2005), and (ii) The globally averaged wavenumber spectrum for the observed SSH has a slope between $-3$ and $-4$ (Stammer 1997). The contours of constant energy and constant frequency, the latter based on the dispersion relation, $\omega_k = -\beta k_x/(k_x^2 + k_y^2 + (f/c)^2)$, in the wavenumber
plane are illustrated in Fig. 13a. For simplicity, we consider a 10000 km by 10000 km square domain upon which the wavenumbers $k_x$ and $k_y$ are non-dimensionalized. The parameters $f$ and $\beta$ are held constant using their value at 40N, and phase speed $c$ is fixed at 3 $\text{cms}^{-1}$, a rough estimate of the basin-averaged value from Chelton et al. (1998). (The frequency plot in Fig. 13a is fashioned after Fig. 12.1 of Gill 1982, where more background can be found.) Under this setting, the first baroclinic Rossby modes exist only with a period longer than 37 weeks. Thus, our model only deals with the behavior of the low-frequency part of the flow. In Fig. 13a, low-pass filtering with a cut-off frequency $\omega_0$ is equivalent to cutting out the shaded area enclosed by the solid contour for $\omega = \omega_0$.

As such, the filter removes meridionally elongated modes (those with $k_x > k_y$) but spares zonally elongated modes, effectively increasing the zonal anisotropy of the flow field.

The values of $\alpha$ for this system as a function of $T$, the cut-off period for the low-pass filter, are shown in Fig. 13b for $p = 3$ (dashed) and $p = 3.5$ (solid). The calculation of $\alpha$ is done numerically in spectral space with a truncation at wavenumber 300 for both $k_x$ and $k_y$. The values of $\alpha$ are somewhat sensitive to the parameter $p$. The observed value of $\alpha$ at the surface for the North Pacific (from Fig. 3a) for $T = 100$ weeks is indicated in Fig. 13b as the horizontal dashed line. This is not meant to be a comprehensive comparison but to show that the simple model at least produces the asymptotic (at large $T$) value of $\alpha$ that is comparable in magnitude with the observed value. A comprehensive appraisal of this idea must eventually include all major types of waves and vertical modes.

Acknowledgments. The work performed by HPH, AK, and EC for this paper was supported in part by the Office of Naval Research, award N00014-05-1-0492. AK was supported by NASA through membership in its Ocean Surface Topography Science Team.
NM was partly supported by NSF grant OCE05-50853, NASA through membership in its Ocean Surface Topography Science Team, and Japan Agency of Marine-Earth Science and Technology (JAMSTEC) through its sponsorship of IPRC. LDEO contribution number xxxx. IPRC/SOEST publication xxx/xxxx.

References


Figure 1. (a) A randomly selected weekly map of the zonal component of the surface geostrophic velocity derived from AVISO altimetry SSH anomaly. (b) Same as (a) but for the meridional component. Color scales are ± 2, 4, 8, 16 cm s\(^{-1}\) with red and blue indicating positive and negative values, respectively.
Figure 2. Same as Fig. 1 but for a randomly selected 50-week averaged surface geostrophic velocity field. (a) Zonal component. (b) Meridional component. The color scales are $\pm 1, 2, 4, 8 \text{ cms}^{-1}$. 
Figure 3. The degree of anisotropy, $\alpha$, calculated from the surface geostrophic velocity field for (a) The North Pacific, (b) Box 1, (c) Box 2, and (d) Box 3. The geostrophic velocity is derived from altimetry SSH. The numbers on the abscissa indicate that the statistics of $\alpha$ are constructed from 1-week, 5-week, ..., 100-week averaged data. The circles and vertical sticks are mean and standard deviation.
Figure 4. The velocity fields from the ocean model simulation. (a) The long-term (12-yr) mean of the surface zonal velocity. (b) A selected 1-yr (last year of a 20-yr run) mean of the surface zonal velocity. (c) The 1-yr average of the anomalous surface zonal velocity, defined as (b) minus (a). (d) Same as (c) but for the surface meridional velocity. Color scales are $\pm 2, 4, 8, 16 \text{ cms}^{-1}$ for (a) and (b) and $\pm 1, 2, 4, 8 \text{ cms}^{-1}$ for (c) and (d). Red and blue colors indicate positive and negative values, respectively.
Figure 5. Same as Fig. 3 but for the statistics of $\alpha$ for the surface velocity from the model simulation. The open circles in (a) are repeats of the calculation of $\alpha$ for the North Pacific but with the long-term mean included in the velocity field.
Figure 6. Same as Fig. 4 but for the model simulated velocity fields at 1000 m. (a) Long-term mean of $u$. (b) One-year average of $u$. (c) One-year average of the anomaly of $u$, defined as (b) minus (a). The period chosen for (b) and (c) is the same as that for Fig. 4c. (d) Same as (c) but for the $v$ component. Color scales are $\pm 0.5, 1, 2, 4 \text{ cms}^{-1}$ for all panels. Red and blue indicate positive and negative values, respectively.
Figure 7. Same as Fig. 5 but for the statistics of $\alpha$ derived from the model simulated velocity at 1000 m.
Figure 8. Same as Figs. 3 and 5 but for the statistics of $\alpha$ for the simulated velocity in the Tropics (between 18N and 18S). (a) Surface. (b) 1000 m.
Figure 9. The zonal component of the surface geostrophic velocity derived from the 10-year mean absolute dynamic topography of Maximenko and Niiler (2005). Color scales are $\pm 1, 4, 8, 16 \, cm s^{-1}$. Red and blue indicate positive and negative values, respectively.
Figure 10. (a) Selected curves of $\alpha$ as a function of $T$ plotted in linear scale for (top to bottom) $\gamma^{-1} = 8, 16, 32, \text{ and } 64$ weeks based on Eq. (3). The symbols are the observed and simulated values of $\alpha$ for the Box 1 region from Figs. 3b (open circle), 5b (filled circle), and 7b (triangle). (The filled circle almost coincide with open circle at $T = 100$.) (b) The values of $\gamma^{-1}$ as a function of $T$ using Eq. (4) and the observed and simulated values of $\alpha$ from Figs. 3b, 5b, and 7b. The symbols used here correspond to their counterparts in (a).
**Figure 11.** The model domain and bathymetry used for the ocean model simulation. The gray scales for depth are 0.5, 1, 2, 3, 4, 5, 6, 7 km. Areas that are shallower than 500 m are white.
Figure 12. An illustration of the geometry of the terrain-following vertical levels of the model. Shown is the cross section for the North Pacific along 40N. The scales on the ordinate are in km.
Figure 13.  (a) The setup of an idealized scenario for the increase of $\alpha$ with time averaging discussed in Appendix C. The solid contours are for the frequency of the first baroclinic Rossby mode with a darker shading corresponding to a higher frequency. The dashed contours are for the energy, and $k_x$ and $k_y$ are zonal and meridional wavenumbers. See Appendix C for detail. (b) The change in $\alpha$ with the length of time averaging for the idealized model (valid only for $T > 37$ weeks) for $p = 3$ (dashed) and $p = 3.5$ (solid). The horizontal dashed line indicates the observed value of $\alpha$ at $N = 100$ for the North Pacific taken from Fig. 3a.